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<td><strong>Citation</strong></td>
<td>Cheng, N. S., Law, A. W. K., &amp; Findikakis, A. N. (2000). Oil transport in surf zone. Journal of Hydraulic Engineering, 126(11), 803-809</td>
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OIL TRANSPORT IN SURF ZONE

By Nian-Sheng Cheng¹, Adrian Wing-Keung Law² and Angelos N. Findikakis³

Abstract: Oil transport subjected to nearshore currents in the surf zone is considered using the mass conservation equation. Analytical solutions are obtained that describe the longshore distribution of the oil deposition on the shore as well as the length of the shoreline contaminated by the oil slicks transported in the nearshore environment. The solutions are related to the characteristic longshore and onshore velocities, which are evaluated separately, and the oil-holding capacity of the shoreline. The analysis also shows that the distribution of the oil deposition varies due to entrainment of the oil from shoreline to water. Sample computations for typical shoreline types are provided to illustrate the effect of the oil-holding capacity on the oil deposition. Limitations of the analytical results obtained are finally discussed.

Keywords: Oil transport, Oil spills, Nearshore currents, Surf zone, Shoreline, Oil-holding capacity, Re-entrainment, Breaker line.

INTRODUCTION

The growing concern over oil spills in the marine environment has led to the development of mathematical models for the prediction of the fate and transport of oil spills in water in recent decades. The fate and transport are governed by various hydrodynamic, biological and chemical processes. These processes are complicated particularly when an oil slick interacts with a shoreline.

Early studies on the oil interaction with shoreline are rather preliminary. For example, Ford (1985) simply correlated the length of the affected coastline with the size of oil spill. Such a statistic approach may not be satisfactory according to Seip et al. (1986), who reported that there is no correlation between the shoreline length damaged

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and the amount of oil initially spilled. Alternatively, exponential decay functions are used in some models to describe the shoreline deposition (e.g. Shen and Yapa, 1988).

Recent investigations on the oil shoreline interaction are more comprehensive with increasing physical understanding of the oil transport processes. When simulated in two or three-dimensional numerical models, the transport processes can be represented in an empirical or analytical manner. One of the earlier efforts in this regard is due to Reed et al. (1989), who developed a coastal zone oil spill model called COZOIL to simulate the explicit behavior of the spilled oil before and after the coastal contact. In particular, the oil shoreline interaction that has been often ignored in numerical simulations is also incorporated in their model by representing the deposition and entrainment of oil spillets. Reed and Gundlach (1989) applied this model to the Amoco Cadiz oil spill and validated its performance for the actual spill event where the oil shoreline interaction was particularly important. Two important parameters, the oil-holding capacities and removal coefficients, were defined to quantify the interaction of the oil spillets with shoreline. Their empirical values were provided for seven types of shorelines based on field data (Gundlach, 1987).

Cekirge et al. (1995) suggested that for modeling purposes, whether or not stranded oil returns to the water from beaches must be handled as Monte Carlo event. Law et al. (1998) contributed a recent effort to the simulation of the nearshore transport of oil. To estimate the length of the damaged shoreline, they assumed that the longshore transport of oil is terminated by manmade and natural blockages, or opposing longshore currents. The assumption is likely to be too conservative since the deposition of oil onto the shoreline is totally ignored. Further information on the oil transport can be found in reviews presented by Yapa and Shen (1994), Cekirge et al. (1995) and ASCE Task Committee on Modeling of Oil Spills (1996).

So far, the development of the oil shoreline interaction is primarily through empirical formulation. This is due to the fact that the processes in the field are complex with only limited data available. To further improve the understanding of the interaction processes, an analytical framework with the nearshore hydrodynamic processes would be desirable. This was attempted by Kobayashi and Yapa (1995) in a one-dimensional cross-shore simulation, where the surf beat effect on the oil deposition and the wave damping due to the oil slick were specially investigated. Similarly, this study aims to provide a framework on the two-dimensional oil transport subjected to the nearshore currents in the surf zone. The hydrodynamic
aspects related to the transport processes are treated in an analytical manner. Characteristic velocities included in the analysis are proposed, respectively, in the longshore and onshore directions. Longshore distributions of the oil deposition are obtained including the effects of the re-entrainment and the oil-holding capacity of shoreline. The limitations on the analytical results obtained are also addressed.

MASS CONSERVATION IN SURF ZONE

Simulation of the oil shoreline interaction can be considered as a module in the entire process of oil spill assessment. This module is required when the oil slick transverses the breaker line and interacts with the nearshore currents.

The phenomenon of the oil transport in the surf zone is very complicated because many physical factors are present in the related processes. A comprehensive formulation for the interactions of the various factors is rather difficult because of the lack of quantitative information reported in the literature. The objective of this study is to solely explore the effect of the nearshore currents on the oil transport. The following simplifications are made to enable a preliminary quantitative assessment. First, advection is considered dominant, in comparison with diffusion, in the surf zone for a horizontal two-dimensional oil transport model. Second, the oil spillets are large and of light density so that they mainly float in the surface layer in spite of the vertical mixing in the surf zone. In addition, the seaward oil flux at the breaker line from the surf zone is assumed negligible. Note that these assumptions lead to an obvious simplification of the oil transport under realistic situations. Therefore, when used in practical applications, the analysis would need to be modified by superimposing effects caused by other significant factors. Further discussion in this regard is given in a subsequent section.

With the above considerations, a two-dimensional equation for the mass conservation in the surf zone can be written in the form:

\[
\frac{\partial m}{\partial t} + \frac{\partial um}{\partial x} + \frac{\partial vm}{\partial y} = 0
\]

where \( m \) = mass of oil per unit area; \( t \) = time; \( x \) = coordinate in the offshore direction; \( y \) = coordinate in the longshore direction; \( u \) = shoreward surface velocity; and \( v \) = longshore current velocity (Fig. 1).
Integrating (1) with respect to $x$ from $x = 0$ to $x = x_b$ where $x_b =$ width of the surf zone, one obtains

$$x_b \frac{\partial m}{\partial t} + q_d - q_r + x_b \nu \frac{\partial m}{\partial y} = 0$$

where $q_d$ = rate of the oil to be deposited on the shore per unit length; $q_r$ = oil re-entrainment rate per unit length; and $\nu$ = average longshore velocity in the surf zone. When integrating (1), the average longshore velocity and the width of the surf zone are considered unchanged. Both the deposition rate and the re-entrainment rate appear as the source terms in (2). The oil re-entrainment rate is first ignored in the following analysis. Its effect on the oil deposition distribution on the shoreline is then especially discussed in a subsequent section.

Eq. (2) shows that oil in the surf zone is transported in either the longshore direction or the shoreward direction. In the longshore direction, oil is pre-dominantly transported by the longshore current. The average longshore velocity over the surf zone can thus be used for such a purpose. In comparison, the velocity of the onshore-offshore current driven by the wave action changes its direction from the water surface to the seabed. It is directed onshore near the water surface and offshore near the bottom. However, for the oil spillets that mainly float on the surface, only the surface velocity is significant in the process of the oil transport. Therefore, the deposition rate of the oil on the shoreline can be expressed as

$$q_d = u_s m$$

where $u_s =$ characteristic surface velocity towards the shoreline.

Note that the deposition rate depends not only on the characteristics of the oil and the approaching waves but also on the oil-holding capacity of the shore. The above formulations only concern the case with a large oil-holding capacity. If the oil-holding capacity is limited, the governing equation must be modified to include its effect on the oil transport. This can be done simply by changing (2) to the following form:

$$x_b \frac{\partial m}{\partial t} + u_s m + x_b \nu \frac{\partial m}{\partial y} = 0$$

where
\[ u_{s1} = \begin{cases} 0 & \text{for } M_s = M_a \\ u_s & \text{for } M_s > M_a \end{cases} \] (5)

in which \( M_s = \) oil-holding capacity in mass per unit length for the shore under consideration; and \( M_a = \) mass of the oil accumulated on the shore segment from the first contact of oil with the shoreline to a certain time step under consideration. (4) shows that once the oil-holding capacity is reached, \( u_{s1} = 0 \) and oil slicks will remain in the water and be subjected to the longshore transport.

In the following, methods are first proposed to evaluate the parameters, \( u_s, \) \( v \) and \( M_\ast \) included in (2) and (4). Analytical solutions to (2) and (4) with sample computations are then presented.

**EVALUATION OF PARAMETERS**

*Average Surface Velocity towards Shoreline, \( u_s \)*

In the cross-shore direction, although the depth-averaged mass flux is zero, there is a net flux shorewards in the surface region and an undertow near the bottom driven by the wave action. The shoreward flow can be considered to be comprised of two parts (Svendson, 1984). One is the top layer carried in the roller, and the other is the mass of water moving between the roller and the trough of the wave. If the average surface velocity for the oil moving towards the shoreline is assumed to be proportional to the average velocity of the top layer, it can be expressed in the form

\[ u_s = \alpha \frac{Q_s}{x_b} \int_0^{x_b} \frac{dx}{H} \] (6)

where \( \alpha = \) coefficient; \( Q_s = \) flux due to the surface roller per unit length and \( H = \) wave height. Furthermore, the flux due to the surface roller takes the form (Svendson, 1984):

\[ Q_s = \frac{cA}{L_w} \] (7)

where \( c = \sqrt{gh} \); \( h = \) water depth; \( A = \) area of the roller \( \approx 0.9H^2 \); \( L_w = \) wave length = \( cT \); and \( T = \) wave period. Substituting of (7) into (6) yields

\[ u_s = \frac{0.9\alpha}{x_b} \int_0^{x_b} \frac{H}{T} \, dx \] (8)
Within the surf zone, the wave height can be approximated using the following formula (Anderson and Fredsoe, 1983):

$$ H = \tan \beta \left[ 0.5x + 0.3x \exp \left( 0.11 \frac{x - x_b}{h_b} \right) \right] $$

(9)

where \( \tan \beta \) = slope of the beach; and \( h_b \) = water depth at the breaking point. If the wave period is almost constant in the surf zone, integration of (8) with (9) leads to

$$ u_s = \frac{\alpha h_b}{T} \left[ 0.23 + 2.46 \tan \beta - 22.31 \tan^2 \beta \left( 1 - \exp \left( - \frac{0.11}{\tan \beta} \right) \right) \right] $$

(10)

(10) shows that the average surface velocity is dependent on the beach slope, the wave period, and the water depth at the breaking point. It is this shoreward velocity that causes the oil slicks floating in the surf zone to move onshore.

**Average Velocity of Longshore Current, \( \bar{v} \)**

The average velocity of the longshore current across the surf zone \((x = 0 \text{ to } x_b)\) can be defined as

$$ \bar{v} = \frac{1}{x_b} \int_0^{x_b} v dx $$

(11)

where \( v \) = longshore current velocity. For convenience, (11) can also be written in the following dimensionless form:

$$ \frac{\bar{v}}{v_b} = \int_0^1 V dX $$

(12)

where

$$ V = \frac{v}{v_b} $$

(13)

$$ X = \frac{x}{x_b} $$

(14)

and \( v_b \) = velocity of the longshore current at the breaker line. Various formulae have been proposed for predicting the time-averaged longshore current velocity (see Horikawa, 1988). One of the well-cited formulae was derived by Longuet-Higgins (1970) based on the concept of the radiation stress. Using his result, \( v_b \) can be expressed in the form
\[ v_b = \frac{5\pi K}{16} f \sqrt{gh_b} \tan \beta \sin \alpha_b \] (15)

where \( K \) = ratio of the wave height to the mean water depth; \( f \) = bottom friction factor; and \( \alpha_b \) = wave angle at the breaker line. The relationship between \( V \) and \( X \) within the surf zone is expressed by Longuet-Higgins (1970) in the form:

\[ V = \frac{1}{1-2.5P} \left( X + \frac{P_2-1}{P_1-P_2} X^P \right) \] (16)

where

\[ P_1 = -\frac{3}{4} + \frac{9}{16} + \frac{1}{P} \] (17)

\[ P_2 = -\frac{3}{4} - \frac{9}{16} + \frac{1}{P} \] (18)

The dimensionless parameter \( P \) represents the relative strength of the lateral mixing to the bottom friction and can be expressed in the form:

\[ P = \frac{\pi N \tan \beta}{Kf} \] (19)

where \( N = 0 \sim 0.016 \). Integration of (16) from \( X = 0 \) to \( X = 1 \) yields the relative average velocity of the longshore current over the width of the surf zone:

\[ \bar{V} = \frac{\bar{v}}{v_b} = \frac{1}{1-2.5P} \left( \frac{1}{2} + \frac{P_2-1}{P_1-P_2} \frac{1}{P_1+1} \right) \] (20)

Fig. 2 shows the relationship between \( V \) and \( P \) based on (20). The relative average velocity is equal to 0.5 when \( P \) approximates 0, where the longshore current velocity is linearly distributed across the surf zone. The relative average velocity decreases rapidly when \( P \) increases. Note that there is a singularity at \( P = 0.4 \) in the relationship of \( V \) and \( P \) according to (16). This can be avoided simply by fitting (20) to an equation in the form:

\[ \bar{V} = \frac{\bar{v}}{v_b} = \frac{2}{4 + 6.5P^{0.75}} \] (21)

Fig. 2 shows that the difference between (20) and (21) is acceptable. Therefore, (21) can be used to compute the average velocity of the longshore current across the surf zone if the longshore current velocity at the breaker line \( v_b \) and the dimensionless parameter \( P \) are known.
Oil-Holding Capacity of Shorelines, $M_*$

The maximum amount of oil that can be held by a beach segment is dependent on the oil and beach characteristics. It consists of two components: maximum surface loading and maximum subsurface loading (Gundlach, 1987; Reed et al., 1989). The maximum surface loading can be represented using the maximum surface oil thickness, $T_m$, and the tidal range including the swash zone, $L_t$. The maximum subsurface oil loading is related to the depth of the oil penetration, $D_p$, the oil content of the sediment, $C_v$, and the width of the swash zone, $L_s$. Therefore, the oil-holding capacity can be expressed as

$$M_* = \rho_o(L_T T_m + C_v D_p L_s)$$  \hspace{1cm} (22)

where $\rho_o$ = density of oil. As general approaches are not available for evaluating the parameters, $T_m$, $C_v$ and $D_p$, the empirical values derived by Gundlach (1987) from the field data for these parameters are instead used in this study.

To derive the values, seven types of commonly encountered shorelines were identified (Gundlach, 1987; Reed et al., 1989). They are rocky cliff, sandy beach, gravel beach, tidal flat, rocky shore, marsh and eroding peat scarp. In addition, based on magnitudes of the viscosity of oil, Gundlach (1987) categorized the oil into three types: low-viscosity (less than 30 cs); mid-viscosity (30-2000 cs); and high-viscosity (greater than 2000 cs). For the seven types of shorelines and three types of oil, Table 1 summarizes the empirical values obtained by Gundlach (1987) for the maximum thickness of surface oil, the depth of oil penetration and the volumetric oil content.

LONGSHORE DISTRIBUTION OF OIL DEPOSITION

Consider that an initial oil slick is released in the surf zone at $y = 0$. In the presence of the nearshore currents, the oil is transported in the longshore current direction in addition to its deposition on the shoreline. Such a process will finally cause a non-uniform distribution of the oil deposition along the shoreline. If the oil-holding capacity of the shoreline is large, the oil can always be allowed to deposit along the shoreline. On the other hand, the oil slick can be continuously transported downstream when the oil-holding capacity is reached.

If the oil-holding capacity is large compared to the possible amount of the oil deposition, a longshore distribution of the oil deposition can be obtained analytically...
by solving (2). Assume that an oil slick with a total mass, $M_o$, is initially released at $y = 0$. For $y > 0$, integrating (2) with respect to $t$ and ignoring the re-entrainment rate yield
\[
\int_0^\infty mdt + x_b \bar{V} \frac{\partial}{\partial y} \left( \int_0^\infty mdt \right) = 0
\]
Eq. (23) can easily be solved as
\[
\int_0^\infty mdt = B \exp \left( - \frac{u_s y}{x_b \bar{V}} \right)
\]
where $B = \text{constant}$. Eq. (24) shows that as the slick moves downstream, the total accumulated mass at a surf zone segment decreases exponentially with the distance $y$. The constant $B$ can be determined based on the fact that the total mass passing through the section $y = 0^+$ must be equal to $M_o$, i.e.
\[
M_o = \int_{-\infty}^\infty x_b \bar{V} mdt \bigg|_{y=0^+}
\]
Substituting (24) into (25) and solving $B$ gives
\[
B = \frac{M_o}{x_b \bar{V}}
\]
Using (24) and (26), the deposited mass on the shoreline per unit length can thus be expressed as
\[
S = \int_0^\infty u_s mdt - \frac{M_o}{x_b \bar{V}} u_s \exp \left( - \frac{u_s y}{x_b \bar{V}} \right)
\]
Eq. (27) shows the longshore distribution of the oil mass deposited on the shoreline. Obviously, integration of (27) from $y = 0$ to $y = \infty$ yields the total oil mass, $M_o$, showing that the initially released amount is fully deposited on the beach. This is because oil losses caused by other processes such as evaporation and bio-degradation are not incorporated.

Furthermore, consider that the initial slick is of finite size and distributed uniformly over an area ranging from $x = 0$ to $x = x_b$ and from $y = 0$ to $y = L$. The total mass $M_o$ is then equal to $m_o L x_b$, where $m_o = \text{mass of the oil per unit area}$. With this condition, the deposition profile can be obtained by integrating (27) in the form
\[
S = \frac{M_o}{x_b \bar{V}} u_s \frac{1}{L} \int_0^L \phi \left( y - \zeta \right) \exp \left( - \frac{u_s \left( y - \zeta \right)}{x_b \bar{V}} \right) d\zeta
\]
where \( \phi(\ y - \zeta \ ) = \) Heaviside step function defined as follows:

\[
\phi(\ y - \zeta \ ) = \begin{cases} 
1 & \text{for } \ y - \zeta \geq 0 \\
0 & \text{for } \ y - \zeta < 0 
\end{cases}
\]  

Eq. (28) can be integrated as:

\[
\frac{SL}{M_o} = \left[ \exp\left(-q \frac{y-L}{L}\right) - 1 \right] \phi\left(\frac{y-L}{L}\right) - \exp\left(-q \frac{y}{L}\right) + 1
\]  

where \( q = (u, L)/(v x_b) \), representing the intensity of the onshore current relative to the longshore current. Eq. (30) shows that the dimensionless mass deposited per unit length, \( SL/M_o \), is dependent on the parameter \( q \) and the dimensionless distance \( y/L \).

Using (30), different distributions of the oil deposited on the shoreline can be computed for different longshore current velocities. Such an example is given in Fig. 3, in the case of \( x_b/L = 0.5 \). The results show that, as expected, the length of the contaminated shoreline is short for the low longshore current velocity, while it is increased for the high longshore current velocity.

To quantify the length of the contaminated shoreline, it is necessary to know the accumulated mass of the oil deposited along the shoreline. The latter is defined as

\[
S_{ca} = \int_{0}^{y} Sdy
\]  

By integrating (30) from \( y = 0 \) to \( y = y \), one obtains

\[
\frac{S_{ca}}{M_o} = \frac{y}{L} \left[ 1 - \exp\left(-q \frac{y}{L}\right) \right] + \phi\left(\frac{y-L}{L}\right) \left[ 1 - \frac{y}{L} + \frac{L}{q} \left[ 1 - \exp\left(-q \frac{y-L}{L}\right) \right] \right]
\]  

Using (32), variations of \( S_{ca}/M_o \) with \( y/L \) are plotted in Fig. 4. It shows that the length of the contaminated shoreline obviously varies with the relative intensity of the onshore current. It changes from 1.1\( L \) for \( q = 10 \), to 6.5\( L \) for \( q = 0.5 \), and to 30.5\( L \) for \( q = 0.1 \) for the case that 95\% of the initially released oil slick is deposited on the shoreline, i.e., \( S_{ca}/M_o = 0.95 \).

**Effect of Re-entrainment Rate**

The deposition profile derived previously is only suitable for areas with low re-entrainment rates. When the re-entrainment rate of the oil is considered, the profile of the oil deposition on the shoreline changes and the length of the contaminated shoreline is expected to increase.
The re-entrainment of the oil is related to the amount of the deposited oil, the wave energy and the tidal condition. It increases with increasing oil deposition and wave energy and during the tidal flood period. It would also depend on shoreline type. For example, salt marshes and mangroves would have lower oil re-entrainment than exposed rocky shores. Other than the empirical decay model of Gundlach (1987), discussion of the physics of oil re-entrainment is glaringly missing in the literature so far.

In this study, to simplify the formulation, the amount of the oil re-entrainment from a shore segment is assumed to be proportional to the oil originally deposited on the segment. A coefficient of proportionality, $C_r$, is thus defined as the ratio of the oil amount re-entrained to that originally deposited. This re-entrainment coefficient varies, as expected, depending on the shoreline type, and wave and tidal conditions. In other words, we assume

$$\int_0^\infty q_r dt = C_r \int_0^\infty q_o dt$$  \hspace{1cm} (33)

where $C_r$ = re-entrainment coefficient. Substituting (3) and (33) into (2), and then integrating (2) with respect to $t$ for $y > 0$, one obtains

$$(1 - C_r) u \int_0^\infty m dt + x_v \bar{v} \frac{\partial}{\partial y} \left( \int_0^\infty m dt \right) = 0$$ \hspace{1cm} (34)

This is the same as the previous governing equation (23) except that the shoreward flux is reduced by a coefficient $(1 - C_r)$. Therefore, the previous solutions obtained for the oil deposition profile and the accumulated oil deposition distribution, i.e., (30) and (32), can be used provided that the parameter $q$ is replaced with $(1 - C_r) q$. Fig. 5 shows the longshore deposition profiles with different re-entrainment coefficients for $q = 0.5$. The results indicate that the larger the re-entrainment coefficient, the longer the length of the contaminated shoreline. Fig. 6 shows the relationship between the dimensionless accumulated oil deposition $S_{cu}/M_o$ and the dimensionless distance $y/L$ for $q = 0.5$. If 95% of the initially released oil mass is deposited on the shore, the computed length of the shoreline contaminated increases from $6.5L$ for $C_r = 0$ to $10.5L$ for $C_r = 0.4$, and to $15.5L$ for $C_r = 0.6$. 
Effect of Oil-Holding Capacity

The oil deposition along the shoreline should also be modified if the effect of the oil-holding capacity is included. This is because the oil-holding capacity is generally limited in practice. Such a modification is sketched in Fig. 7 and it can be implemented by applying the following procedures.

1. First, (30) is used to determine the oil deposition distribution in the case of large oil-holding capacity, as shown in Fig. 7(a).

2. Using (30), find out the maximum value of the oil deposition distribution, i.e.
   \[ S_{\text{max}} = \frac{M_o}{L} [1 - \exp(-q)] \]  
   If \( S_{\text{max}} \) is greater than the oil-holding capacity, \( M_* \), then the longshore distribution of the oil deposition needs to be modified.

3. Compute \( y_1 \) and \( y_2 \) using the following equations:
   \[ M_* = \frac{M_o}{L} \left[ 1 - \exp \left( -\frac{q y_1}{L} \right) \right] \]
   \[ M_* = \frac{M_o}{L} \left[ \exp \left( -\frac{q y_2}{L} + q \right) - \exp \left( -\frac{q y_2}{L} \right) \right] \]
   Both (36) and (37) are derived from (30).

4. The \( y_3 \)-value is determined, based on the mass conservation, in the form:
   \[ y_3 = \frac{M_o - (M_1 + M_2)}{M_*} + y_1 \]
   where \( M_1 \) = accumulated mass from \( y = 0 \) to \( y = y_1 \), and \( M_2 \) = accumulated mass from \( y = y_2 \) to \( y = \infty \). With reference to the modified longshore distribution as sketched in Fig. 7(b), \( y_3 \) is the location after which \( S \) is always less than \( M_* \). Both \( M_1 \) and \( M_2 \) are computed based on the oil deposition distribution for large oil-holding capacity. Therefore, with (30), they can be defined, respectively, as
   \[ M_1 = \int_0^{y_1} S \, dy = \frac{M_o}{q} \left[ \frac{q y_1}{L} + \exp \left( -\frac{q y_1}{L} \right) - 1 \right] \]
   \[ M_2 = \int_{y_2}^{\infty} S \, dy = \frac{M_o}{q} \left[ \exp \left( -\frac{q y_2}{L} + q \right) - \exp \left( -\frac{q y_2}{L} \right) \right] \]

5. Finally, the modified longshore distribution of the oil deposition can be expressed as
The five typical kinds of shorelines as mentioned previously are used to illustrate the effect of the oil-holding capacity on the oil deposition distributions. The properties of the shorelines including the surface distance and swash zone distance as shown in Table 2 are based on Gundlach (1987). The oil-holding capacity for each shoreline is evaluated using (22) and Table 1. For example, the volumetric capacity is 0.01 m$^3$/m for the rocky shore with a surface distance of 5 m; and it increases to 1.75 m$^3$/m for the sandy beach with a surface distance of 100 m.

For computational purposes, the shoreline is divided into segments with a suitable length. The onshore oil accumulation in each segment is checked to see if the oil-holding capacity of the particular shoreline type is reached. The conditions selected for the computation are $M_o = 10$ m$^3$, $L = 5$ m, $x_o = 10$ m and $u_o/v = 1$. They are chosen to exemplify the effect of the oil-holding capacities on the oil distribution. Note that the selected values for the total mass of the oil released and the ratio of the onshore to longshore velocities may be much larger than those usually encountered in the field conditions.

For the same amount of the initially released oil mass, Fig. 8 shows the different deposition distribution profiles modified for the five different shoreline types. In the case of the sandy beach, the longshore deposition profile does not change as its oil-holding capacity is larger than $S_{max}$ defined by (35). Similarly, there is only a slight modification in the profile for the gravel beach. On the other hand, the changes are significant for the shoreline types like the marsh, tidal flat and rocky shore. This is because the oil-holding capacities for these shorelines are much less than $S_{max}$ defined by (35) and more oil slicks undergo a longer period of longshore transport prior to the onshore deposition. Fig. 9 shows the accumulated oil deposition profiles for the five shorelines. The oil-affected longshore length for each shoreline is evaluated, as shown in Table 2, for the case that the accumulated mass of oil deposited reaches 95% of the initial amount, i.e. $S_{cum}/M_o = 95%$. The affected shoreline length varies slightly from 32.5 m to 42 m for the sandy beach, gravel beach and marsh. However, it increases...
rapidly to 82.5 m for the tidal flat, and 950 m for the rocky beach because of the low oil-holding capacities.

**DISCUSSION**

The simplicity of the analysis performed in this study gives some insight into the fundamental mechanisms of the oil transport in the surf zone. The analytical results obtained can be easily included as a module in the entire process of oil spill assessment. However, it should be noted that many other factors in addition to the nearshore currents considered here could be important in a realistic situation of the oil transport in the surf zone. The effects of these factors on the oil transport may significantly modify the results obtained.

For example, an onshore current driven by wind stress is observed to be a favorable factor for the oil deposition. In addition, tidal effect may be important as the oil deposition mostly occurs during the ebb period (see Reed et al., 1989). In the vertical water column, mixing can be strengthened through turbulence so that the oil spillets can interact directly with sediment near the seabed. Particularly, when the oil spillets are fine and the buoyant velocities are small, this may lead to a near uniform distribution of the oil throughout the water column, and hence a high concentration near the bed. In other words, ignoring the vertical mixing in the water column may reduce the amount of oil mass present in the surface layer of water, which is thus subjected to the onshore current. Furthermore, tidal currents in the surf zone can be dominant for some cases in comparison with the currents generated by the waves. Such factors would have significant contributions to the oil transport in the surf zone.

A direct comparison between the present analysis with field data is almost impossible due to the simplifications. On the other hand, despite the approximations taken and the associated limitations, the study provides a relatively rigorous approach towards the treatment of the hydrodynamics involved in the oil transport. This approach can be further improved by including the physico-chemical interaction processes in the future, when more information on the oil shoreline interaction is available.
CONCLUSIONS

Oil transport in the surf zone is investigated by considering the effect of the average nearshore currents. Analytical solutions are obtained to describe the longshore distributions of the onshore oil deposition as well as to estimate the oil-affected length of the shoreline. The results show that the length of the affected shoreline is much longer when the oil-holding capacity and the intensity of the relative onshore current are decreased. The effects of the re-entrainment and the oil-holding capacity on the oil deposition distribution are further illustrated using the sample calculations.

APPENDIX I. REFERENCES
transport.” Proc. of International Conference on Computer Applications in Environmental Engineering, Malaysia.


**APPENDIX II. NOTATION**

*The following symbols are used in this paper:*

\[ A = \text{area of the roller} \approx 0.9H^2; \]

\[ B = \text{constant}; \]

\[ C_v = \text{oil content of the sediment}; \]

\[ D_p = \text{depth of oil penetration}; \]

\[ C_r = \text{re-entainment coefficient}; \]

\[ c = \sqrt{gh}; \]

\[ f = \text{bottom friction factor}; \]

\[ H = \text{wave height}; \]

\[ h = \text{water depth}; \]

\[ h_b = \text{water depth at the breaking point}; \]

\[ K = \text{ratio of the wave height to the mean water depth}; \]

\[ L = \text{longshore length of the initial oil slick}; \]

\[ L_s = \text{width of the swash zone}; \]
\( L_t \) = tidal range including the swash zone;
\( L_w \) = wave length = \( cT \);
\( M_1 \) = accumulated mass from \( y = 0 \) to \( y = y_1 \);
\( M_2 \) = accumulated mass from \( y = y_2 \) to \( y = \infty \);
\( M_d \) = mass of the oil accumulated on the shore segment;
\( M_o \) = total amount of the initial oil slick = \( m_o L x_b \);
\( M_o^* \) = oil-holding capacity in mass per unit length;
\( m \) = mass of oil floating in water per unit area;
\( m_o \) = mass of the water-carrying oil per unit area at \( t_1 = 0 \);
\( N \) = constant;
\( P \) = dimensionless parameter;
\( P_1, P_2 \) = functions of \( P \);
\( Q_s \) = flux due to the surface roller per unit length;
\( q \) = \( (u, L)/(v x_b) \);
\( q_d \) = rate of the oil to be deposited on the shore per unit length;
\( q_r \) = oil re-entrainment rate;
\( S_o \) = original deposition on the shore per unit length;
\( S_{sa} \) = accumulated mass of the oil deposited along the shoreline;
\( S_{max} \) = maximum value of the oil deposition distribution;
\( T \) = wave period;
\( T_m \) = maximum surface oil thickness;
\( t \) = time;
\( u \) = shorewards surface velocity;
\( u_s \) = characteristic surface velocity towards the shoreline;
\( u_{sl} \) = characteristic onshore velocity;
\( V \) = \( v / v_b \);
\( \overline{V} \) = \( \bar{v} / v_b \);
\( v \) = longshore current velocity;
\( v_b \) = velocity of the longshore current at the breaker line;
\( \bar{v} \) = average longshore velocity in the surf zone;
\( X \) = \( x / x_b \);
\( x \) = coordinate in the offshore direction;
\( x_b \) = width of the surf zone;
\( y \) = coordinate in the longshore direction;
\( y_1, y_2, y_3 \) = characteristic positions shown in Fig. 5;
\( \alpha \) = coefficient;
\( \alpha_0 \) = wave angle at the breaker line;
\( \beta \) = angle of beach slope;
\( \zeta \) = variable of integration;
\( \rho_o \) = density of oil; and
\( \phi \) = Heaviside step function.
Captions for Figures

Fig. 1. Definition Sketch of Oil Movement in Surf Zone.

Fig. 2. Variation of Average Longshore Velocity with P.

Fig. 3. Distribution of Oil Deposited on Shoreline for Different Ratios of Current Velocities.

Fig. 4. Relationship of Accumulated Oil Deposition with Contaminated Shoreline Length.

Fig. 5. Effect of Re-entrainment on Oil Deposition Distribution.

Fig. 6. Accumulated Oil Deposition with Different Re-entrainment Coefficients.

Fig. 7. Modification of Oil Deposition Distribution for Low Oil-Holding Capacity.

Fig. 8. Longshore Profiles of Oil Deposition for Typical Shorelines.

Fig. 9. Distributions of Accumulated Oil Deposition for Typical Shorelines.
Table 1. Oil Holding Capacities for Different Shoreline Types

<table>
<thead>
<tr>
<th>Shoreline type</th>
<th>Maximum surface oil thickness, $T_m$ (mm)</th>
<th>Subsurface oil holding capacity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Light viscosity</td>
<td>Medium viscosity</td>
</tr>
<tr>
<td>Rocky cliff</td>
<td>0.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Sandy beach</td>
<td>4</td>
<td>17</td>
<td>25</td>
</tr>
<tr>
<td>Sand and gravel beach</td>
<td>2</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>Tidal flat</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Rocky shore</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Marsh</td>
<td>6</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>Eroding peat scarp</td>
<td>1</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>
Table 2. Affected Length of Shoreline for Different Oil-Holding Capacities

<table>
<thead>
<tr>
<th>Shoreline type</th>
<th>Rocky shore</th>
<th>Sandy beach</th>
<th>Gravel beach</th>
<th>Tidal flat</th>
<th>Marsh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface distance, $L_s$(m)</td>
<td>5</td>
<td>100</td>
<td>50</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Oil thickness, $T_m$(m)</td>
<td>0.002</td>
<td>0.017</td>
<td>0.009</td>
<td>0.006</td>
<td>0.030</td>
</tr>
<tr>
<td>Swash zone distance, $L_s$(m)</td>
<td>-</td>
<td>10</td>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Oil penetration, $D_p$(m)</td>
<td>-</td>
<td>0.05</td>
<td>0.18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Content (%)</td>
<td>-</td>
<td>9.8</td>
<td>8.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Oil-holding capacity, $M$(m$^3$/m)</td>
<td>0.01</td>
<td>1.75</td>
<td>0.60</td>
<td>0.12</td>
<td>0.30</td>
</tr>
<tr>
<td>Affected shoreline length for $S_{cu}/M_o = 95%$, (m)</td>
<td>950.0</td>
<td>32.5</td>
<td>33.0</td>
<td>82.5</td>
<td>42.0</td>
</tr>
</tbody>
</table>
$S_{\text{cu}}/M_0$ vs $y/L$ for different $C_r$ values:
- $C_r = 0.0$
- $C_r = 0.2$
- $C_r = 0.4$
- $C_r = 0.6$